Readers' Forum

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Comment on "Hamilton's Law and the Stability of Nonconservative Continuous Systems"

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PROFESSOR Bailey[1] reports some interesting numerical work in his Note. However, there are several errors in concept and mechanics which should be pointed out.

His misconceptions concerning nonconservative systems and work functions have been previously pointed out in his Refs. 2 and 3. Nothing more need be or can be said at this time, except to wonder why he insists on associating work functions with nonconservative systems when a correct formulation is so well-known, is so readily available, does not require the concept of a restricted variation of a functional, and gives exactly the same results which he quotes.

In the formulation of the beam problem, Bailey states that plane sections remaining plane implies $v=v_0-z(\partial w/\partial y)$. This is of course not true. His expression for v also requires the assumption that the transverse shear strain is zero. But in that case there would be no need for the expression γ_{yz}. Also note that γ_{yz} contains sign errors in the nonlinear terms if Bailey wishes to use the Green strain tensor as indicated by his expression for ϵ_{yy}.

Equation (4) is not complete, which leads to an error of interpretation in Eq. (5). This can be most easily demonstrated through a brief virtual work derivation of the equations. Consider the cantilever beam with end loads and distributed axial loading. Assume a uniaxial stress state and Kirchhoff hypotheses for deformation. Then in Bailey's coordinate system, the principle of virtual work says

$$-\int_0^\ell \int_A \left(\rho\ddot{v}\delta v+\rho\ddot{w}\delta w\right)\mathrm{d}A\,\mathrm{d}y+\int_0^\ell \bar{S}_y\delta v_0\,\mathrm{d}y$$
$$+P_y\delta v_0\Big|_\ell+P_z\delta w\Big|_\ell-\int_0^\ell\int_A \sigma_{yy}\delta\epsilon_{yy}\,\mathrm{d}A\,\mathrm{d}y=0$$

where ρ is the density of the beam material. Substitute the assumed forms for the displacement components and perform the integration over the cross-sectional area A. The result is

$$-\int_0^\ell\left[(\rho A\ddot{v}_0-\bar{S}_y)\delta v_0+N\frac{\partial\delta v_0}{\partial y}\right]\mathrm{d}y+P_y\delta v_0\Big|_\ell$$
$$-\int_0^\ell\left[\rho A\ddot{w}\delta w+\left(\rho I\frac{\partial^3 w}{\partial t^2\partial y}+N\frac{\partial w}{\partial y}\right)\frac{\partial\delta w}{\partial y}\right.$$
$$\left.+M\frac{\partial^2\delta w}{\partial y^2}\right]\mathrm{d}y+P_z\delta w\Big|_\ell=0 \tag{1}$$

where

$$N\equiv EA\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right],\qquad M\equiv EI\frac{\partial^2 w}{\partial y^2}$$

Now, a single time integration by parts, with suitable restrictions on δv_0, will demonstrate that Bailey's Eq. (3) is equivalent to the vanishing of the first line of Eq. (1) above. If the axial acceleration $\ddot{v}_0$ is neglected, then it follows that $\partial N/\partial y+\bar{S}_y=0$ in the column domain. If $\bar{S}_y=Y_1+Y_2y$, then

$$N=P_y+Y_1(\ell-y)+\tfrac{1}{2}Y_2(\ell^2-y^2)$$
$$=\ell\left[\frac{P_y}{\ell}+Y_1+\tfrac{1}{2}Y_2\ell-Y_1\left(\frac{y}{\ell}\right)+Y_2\ell\left(\frac{y}{\ell}\right)^2\right]$$

Therefore, the right hand side of Bailey's Eq. (5), with the obvious changes necessary to achieve correct dimensions, provides the result for the axial strain $\epsilon_{yy}=\partial v_0/\partial y+\tfrac{1}{2}(\partial w/\partial y)^2$. Note that in general $(\partial w/\partial y)^2$ is not negligible compared to $\partial v_0/\partial y$ since the essence of the geometrically nonlinear theory is that these two terms are of the same order.

The vanishing of the second and third lines of Eq. (1) above supplies the transverse equilibrium requirements. Note that Bailey's Eq. (4) does not include the rotary inertia term, which may have been omitted because shear deformation was ignored and with the knowledge that rotary inertia would be neglected in the numerical study. Also it can be seen that the term $N(\partial w/\partial y)(\partial\delta w/\partial y)$ is not properly represented due to neglect of the nonlinear term in the axial strain-displacement relation. In other words, in both Eqs. (4) and (5), Bailey has implied that the axial force is equal to $EA\partial v_0/\partial y$. This error in formulation is of no consequence in the reported numerical results since the fundamental state of equilibrium is straight. However, the error would affect results with bending in the fundamental state.

The final comment is that the Beck problem has been solved many times before through approximate methods. References 2 and 3 are particularly appropriate since both authors use the generalized Hamilton's principle; Ref. 2 is precise in treatment of the restricted variation caused by nonconservative forces.

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References

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[3] Mote, C. D., Jr., "Nonconservative Stability by Finite Element," *J. Eng. Mech. Div. ASCE,* Vol. 97, No. EM3, June 1971, pp. 645-666.